II. **Solution by J. C. CORBIN, Pine Bluff, Arkansas; P. S. BERG, Larimore, North Dakota; E. W. MORRELL, Montpelier Seminary, Montpelier, Vermont; A. P. READ, A. M., Clarence, Missouri; and O. S. WESTCOTT, Principal North Chicago High School, Chicago.**

Put $N=nD+1$, then it is evident that if $N=nD+1$ be raised to any positive integral power, the last term will be 1 and every other term will contain D as a factor; hence if this power be divided by D the remainder will be 1.

Also solved in a similar way by *A. H. BELL*, *JOSIAH H. DRUMMOND*, *ARTEMAS MARTIN* and *J. SCHEFFER.*

III. **Solution by J. O. MAHONEY, B. E., M. S., Graduate Fellow and Assistant in Mathematics, Vanderbilt University, Nashville, Tennessee.**

If $a\equiv a'$, $b\equiv b'$, $c\equiv c'$, $d\equiv d'$, etc., mod(D),

then $abcd......\equiv a'b'c'd'......mod(D)$.

Let $a=b=c=d$, etc.,$=N$, and $a'=b'=c'=d'$, etc.,$=1$, then $N^k\equiv1$ mod(D).

AVERAGE AND PROBABILITY.

Conducted by B. F. FINKEL, Springfield, Mo. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

39. **Proposed by O. W. ANTHONY, M. Sc., Professor of Mathematics, Columbian University, Washington, D. C.**

A man is at the center of a circular desert; he travels at a given rate but in a *perfectly* random manner. What is the probability that he will be off the desert in a given time?

No solution of this problem has been received.

40. **Proposed by HENRY HEATON, M. Sc., Atlantic, Iowa.**

If every point of an ellipse be joined with every other point, what is the average length of the chords thus drawn?

Solution by the PROPOSER.

Let $a\cos\theta$ and $(b/a)\sin\theta$ be the coördinates of one point, and $a\cos\phi$ and $(b/a)\sin\phi$ those of another.

The length of the chord joining them is

$$K=[a^2(\cos\phi-\cos\theta)^2+\frac{b^2}{a^2}(\sin\phi-\sin\theta)^2]^{\frac{1}{2}}.$$

Let s_1 and s_2=lengths of elliptic arcs from point $(a, 0)$ to points $(a\cos\theta, \frac{a}{b}\sin\theta)$ and $(a\cos\phi, \frac{a}{b}\sin\theta)$ respectively, and let S=whole distance around the ellipse.

Then $\frac{ds_1}{d\theta}=a(1-e^2\cos^2\theta)^{\frac{1}{2}}$ and $\frac{ds_2}{d\phi}=a(1-e^2\cos^2\phi)^{\frac{1}{2}}$.

Then the required average is

$$A=\frac{H}{S^2}\int_0^{\frac{1}{2}\pi}\int_0^{2\pi}K ds_1 ds_2=\frac{4a^2}{S^2}\int_0^{\frac{1}{2}\pi}\int_0^{2\pi}[a^2(\cos\phi-\cos\theta)^2+\frac{b^2}{a^2}(\sin\phi-\sin\theta)^2]^{\frac{1}{2}}\times$$

$$(1-e^2\cos^2\theta)^{\frac{1}{2}}(1-e^2\cos^2\phi)^{\frac{1}{2}}\,d\theta d\phi.$$

This equation cannot be integrated in general terms.

Solved in the same manner by *G. B. M. ZERR*,

41. Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Irving College, Mechanicsburg, Pennsylvania.

A line is drawn at random across the chord and given arc of a circular segment. Find the mean area of the divisions.

Solution by G. B. M. ZERR, A. M., Ph. D., Texarkana, Arkansas-Texas.

Let A=area of given segment, A_1, A_2 mean areas of the two divisions.

$\therefore A_1+A_2=A$.

But, since the line is a random line, $A_1=A_2$.

$\therefore A_1=A_2=\frac{1}{2}A$.

Also solved by *HENRY HEATON*.

42. Proposed by CHARLES E. MYERS, Canton, Ohio.

A attends church 4 Sundays out of 5; B, 5 Sundays out of 6; and C, 6 Sundays out of 7. What is the probability of an event that A and B will be at church and C will not?

Solution by G. B. M. ZERR, A. M., Ph. D., Texarkana, Arkansas-Texas, and B. F. FINKEL, A. M., Professor of Mathematics and Physics, Drury College, Springfield, Missouri.

The chance that A attends church$=\frac{4}{5}$.
The chance that B attends church$=\frac{5}{6}$.
The chance that C attends church$=\frac{6}{7}$.
The chance that A is not at church$=\frac{1}{5}$.
The chance that B is not at church$=\frac{1}{6}$.
The chance that C is not at church$=\frac{1}{7}$.
The chance that A and B attend and C not$=p_1=\frac{4}{5}.\frac{5}{6}.\frac{1}{7}=\frac{2}{21}$.
The chance that A and C attend and B not$=p_2=\frac{4}{5}.\frac{6}{7}.\frac{1}{6}=\frac{4}{35}$.
The chance that B and C attend and A not$=p_3=\frac{5}{6}.\frac{6}{7}.\frac{1}{5}=\frac{1}{7}$.
The chance that A attends and B and C not$=p_4=\frac{4}{5}.\frac{1}{6}.\frac{1}{7}=\frac{4}{210}$.
The chance that B attends and A and C not$=p_5=\frac{1}{5}.\frac{5}{6}.\frac{1}{7}=\frac{1}{42}$.
The chance that C attends and A and B not$=p_6=\frac{1}{5}.\frac{1}{6}.\frac{6}{7}=\frac{1}{35}$.
The chance that A, B and C attend$=p_7=\frac{4}{5}.\frac{5}{6}.\frac{6}{7}=\frac{4}{7}$.
The chance that A, B and C do not attend$=p_8=\frac{1}{5}.\frac{1}{6}.\frac{1}{7}=\frac{1}{210}$.
p_1=probability required.
Also $p_1+p_2+p_3+p_4+p_5+p_6+p_7+p_8=1$.

Also solved by *HENRY HEATON*.